

# Fast numerical algorithms for applying separable integral operators in multiple dimensions

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We have developed and implemented a formalism for the application of separable integral operators in  $d \geq 1$  dimensions using the Non-Standard (NS) form [1]. The proposed formalism is general, compact and oriented towards the practical implementation into a working code using multiwavelets [2]. For the case of Poisson and Helmholtz operators we also propose a simple scheme for the generation of an approximate separated representation of the corresponding kernels with finite arbitrary precision. Such a representation [3, 4], combined with the NS-form, allows us to build a sparse, banded representation of the operator. We have implemented a code for the application of the Poisson and Helmholtz operators on a separated Non-Standard form to a multivariate function. We have also devised a helpful notation for the application of the NS-form which simplify the exposition and the connection to the implementation. The implemented code computes explicitly all the  $2^{2d}$  components of the  $d$ -dimensional operator. We finally demonstrate the performance of our implementation on some test cases.

## REFERENCES

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