Smoothing the payoff of European basket options

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Abstract

The pricing problem of European basket options in a Black-Scholes model leads in certain cases to the calculation of the integral

$$E\left[\left(\sum_{i=1}^{d} w_i e^{X_i} - K\right)^+\right],$$

where $X \sim \mathcal{N}(0, \Sigma)$ with a covariance matrix Σ and $d \gg 1$. Hence, we have to compute an integral over the integration domain $\mathbb{R}^d$ for an integrand with a kink. We provide a simple smoothing technique which produces an analytic integrand and is able to reduce the dimensionality of the integration problem by 1. Moreover, this smoothing does not introduce any approximation error. In particular, we transform the d-dimensional random variable in such a way that it is feasible to apply the famous Black-Scholes formula with respect to a single coordinate. The resulting integration problem over $\mathbb{R}^{d-1}$ of an analytic function is then solved by an adaptive sparse grid approach. This leads, at least in considerably high dimensions, to better convergence results compared to those of standard Monte Carlo or quasi-Monte Carlo quadratures.