An Adaptive Discontinuous Galerkin Method for General Evolution Equations

Andreas Dedner

We present a numeric scheme for solving systems of evolution equations of the general form

$$\partial_t U(t,x) + \nabla \cdot \big(F(U(t,x),t,x) + a(U(t,x),t,x)\nabla U(t,x)\big) = S(U(t,x),t,x)\ .$$

Equation of this form are used for example to model compressible fluid flow. We will focus on the case of convection dominated flow where the viscosity a is assumed to be small compared to the advection forces (modeled by the flux function F). In the case of vanishing viscosity, smooth solutions cannot be expected. If the viscosity is small, steep gradients have to be resolved, which cause stability problems for numerical schemes.

The numerical scheme is based on the higher order *Discontinuous Galerkin* method in space and on implicit-explicit Runge-Kutta methods in time. We use h-adaptivity, i.e., general grid structures with non-conform adaptivity and domain decomposition, to achieve a high degree of efficiency.

Since our focus is on the convection dominated case, we discuss approaches for gradient limiting and p-adaptivity to stabilize the scheme in regions of strong gradients or discontinuities. The basis of our approach is an a-posteriori error estimate for the semi-discrete method.

For the implementation of the scheme we use the software environment DUNE which provides a generic interface for implementing grid based numerical schemes for a wide range of grid structures. Grid adaptivity and parallelization with load balancing are centeral aspects in the construction of efficient numerical schemes for complex applications. The DUNE interface concept provides a simple and transparent approach for using these concepts on distributed memory architectures.